Algebraic Number Theory and Related Topics 2013
Abstracts of talks

December 9th (Mon)

Speaker: Koichi Kawada (Iwate University)
Title: The ternary Goldbach problem — from Hardy-Littlewood, Vinogradov to Helfgott (Survey Talk)
Abstract: In 1472, Goldbach wrote Euler the conjectures that every even integer exceeding 2 is the sum of two primes, and that every odd integer exceeding 5 is the sum of three primes. The latter is called the ternary Goldbach conjecture.

Being based on the circle method that Hardy and Littlewood invented around 1920, Vinogradov essentially resolved the latter conjecture in 1937 by establishing that every odd interger exceeding a certain constant is the sum of three primes. And in May 2013, Helfgott finally succeeded in proving that the ternary Goldbach conjecture is completely true.

The purpose of this survey talk is to illustrate an outline of the proof of Helfgott, for those who are not familiar with the circle method.

Speaker: Yoshinori Mishiba (Kyushu University)
Title: Algebraic independence of certain positive characteristic multizeta values
Abstract: Let $\mathbb{F}_q(\theta)$ be the rational function field over the finite field of q elements with variable θ and $\mathbb{F}_q(\!(\theta^{-1})\!)$ the completion of $\mathbb{F}_q(\theta)$ with respect to the place at infinity. The positive characteristic multizeta values $\zeta(n_1,\dots,n_d)$ are defined in $\mathbb{F}_q(\!(\theta^{-1})\!)$ in the same way as the classical multiple zeta values are defined in $\mathbb{R}$. In this talk, we study algebraic relations over $\mathbb{F}_q(\theta)$ among the elements of $\{\widetilde{\pi}\} \cup \{\zeta(n_i,\dots,n_j) | 1 \le i \le j \le d\}$ under certain conditions on the index $(n_1,\dots,n_d)$, where $\widetilde{\pi}$ is the fundamental period of the Carlitz module. For example, we show that the elements of the above set are algebraically independent over $\mathbb{F}_q(\theta)$ if n_i is not divisible by $q-1$ for each i and n_i/n_j is not a power of the characteristic of $\mathbb{F}_q$ for each $i \neq j$.

Speaker: Koji Tasaka (Kyushu University)
Title: Multiple Eisenstein series and Brown's totally odd multiple zeta values
Abstract: An interesting connection between the theory of elliptic modular forms and multiple zeta values (MZVs) was first discovered by Don Zagier in the case of depth 2 (more extensive results were given by Gangl, Kaneko and Zagier). Recently, Francis Brown has proposed a new conjecture on the dimension of the space spanned by "totally odd" MZVs. His conjecture suggests that the modular forms and the linear relations among totally odd MZVs are deeply related with each other. In this talk, we present an attempt and some results to reveal this relationship using the multiple Eisenstein series.

Speakers: Shin-ichi Yasutomi (Toho University) Jun-ichi Tamura (Tsuda College)
Title: New multidimensional continued fraction algorithms and their applications
Abstract: We give new algorithms of multidimensional continued fraction expansion by which the expansion of $\alpha_0^{-1}(\alpha_1,\dots,\alpha_{d-1})$ is always expected to be periodic for any $\mathbb{Q}$-basis $(\alpha_0,\alpha_1,\dots,\alpha_{d-1})$ of an arbitrarily given real number field K with $d = \deg_{\mathbb{Q}}(K) \le 6$. As applications of our algorithms, we give some numerical results concerning computation of algebraic units and the generation of stepped surfaces that are related to integral points near from the hyperplane $\alpha_0 x_0 + \alpha_1 x_1 + \ldots + \alpha_{d-1} x_{d-1} = 0$ in the Euclidean space $\mathbb{R}^d$.

Speaker: Kazuki Morimoto (Osaka City University)
Title: On algebraicity of special values of L-functions for $\mathrm{SO}(V) \times \mathrm{GL}_2$
Abstract: Let F be a totally real number field, and V a quadratic space over F such that $V \otimes_F F_v$ is positive definite at every archimedean place v of F. Let τ be an irreducible automorphic representation of $\mathrm{SO}(V, \mathbb{A}_F)$. Let f be a holomorphic Hilbert primitive cusp form over F, and we denote by π the irreducible unitary cuspidal automorphic representation of $\mathrm{GL}_2(\mathbb{A}_F)$ associated to f. Then we prove an algebraicity of special values of $L(s, \pi \otimes \tau)$ at various critical points under some assumptions on π_∞ and τ_∞. As a special case, we prove a new algebraicity result for Rankin triple product L-functions for GL_2 in some unbalanced cases. Our algebraicity result conforms with Deligne's conjecture on special values of motivic L-functions at critical points. This is a joint work with Masaaki Furusawa.

December 10th (Tue)

Speaker: Shun'ichi Yokoyama (Kyushu University)
Title: Exploring Modular Forms using Computer Algebra Systems (Survey Talk)
Abstract: We give a brief introduction to the theory of computing modular forms, Galois representations, and relevant structures using a free open-source computer algebra system Sage. We also survey a polynomial time algorithm by Couveignes-Edixhoven-Bruin for computing coefficients of Fourier expansion of modular forms as a recent progression. If time permits, we introduce some related topics of this algorithm.

Speaker: Shin Hattori (Kyushu University)
Title: Ramification theory and perfectoid spaces
Abstract: Consider two complete discrete valuation fields of mixed and equal characteristics and suppose that their integer rings are isomorphic to each other modulo the m-th power of uniformizers. In the '80s, Deligne constructed an isomorphism between their absolute Galois groups modulo the m-th ramification subgroups, if their common residue field is perfect. This enables us to reduce a study of Galois representations of equal characteristic CDVF's to the mixed characteristic case (and vice versa under small ramification). In this talk, I will explain its generalization to the imperfect residue case, using Scholze's theory of perfectoid spaces.

Speaker: Yoshiyasu Ozeki (RIMS, Kyoto University)
Title: On Galois equivariance of homomorphisms between torsion potentially crystalline representations
Abstract: M. Kisin proved that a certain restriction functor on crystalline p-adic representations is fully faithful. We consider the torsion analogue of this result. Such a problem has been shown by C. Breuil for finite flat representations, that is, torsion crystalline representations with (presumable) Hodge-Tate weights in $[0, 1]$. In this talk, we consider higher weight cases.

Speaker: Yuichi Hirano (University of Tokyo)
Title: Congruences of Hilbert modular forms over real quadratic fields and the special values of L-functions
Abstract: Let F be a totally real number field. We consider the problem to show how congruences between the Fourier coefficients of a Hilbert eigenform over F and a Hilbert Eisenstein series over F (of the same parallel weight $(k, \cdots, k)$) give rise to corresponding congruences between the algebraic parts of the special values of the associated L-functions. In the case $F = \mathbb{Q}$, the congruences of L-functions

were obtained by Vatsal ($k = 2$) and the speaker ($k \geq 2$). In this talk, we generalize Vatsal's work to the case F is a real quadratic field and $k = 2$.

Speaker: Yoshitaka Maeda
Title: The so-called Maeda conjecture and related topics
Abstract: I will talk recent progress of the so-called Maeda conjecture on elliptic cusp forms of level 1 and related topics. As related topics, I will explain the followings:
(1) A generalization to cusp forms of higher level by Tsaknias and canonical splittings;
(2) Transformation equations and the special values of Shimura's zeta functions by Doi, Hida and Maeda.

Speaker: Kazuhiro Fujiwara (Nagoya University)
Title: Indivisibility of relative class numbers for quadratic extensions of totally real fields
Abstract: For a totally real field F and an odd prime p, we look for a CM quadratic extension L/F whose relative class number is not divisible by p. This question of indivisibility is classical, and turns out to be fairly difficult. I show the existence of infinitely many L by using modularity and Galois deformations, namely, in the framework of non-abelian class field theory. This is part of my project on non-abelian approaches to classical problems.

December 11th (Wed)

Speaker: Kentaro Nakamura (Hokkaido University)
Title: The p-adic Langlands correspondence I (Survey Talk)
Abstract: In this survey talk, I talk about the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ which was proposed by Breuil. In particular, I talk about the work of Colmez on the construction of the correspondence and the work of Paskunas on the bijectivity of the correspondence. I will talk also about the compatibility with the local Langlands correspondence and the recent developments concerning the generalization to other groups.

Speaker: Sho Yoshikawa (University of Tokyo)
Title: Roots of the discriminant of an elliptic curves and its torsion points
Abstract: We give an explicit and intrinsic description of (the torsor defined by the 12th roots of) the discriminant of an elliptic curve using the group of its 12-torsion points and the Weil pairing. As an application, we extend a result of Coates (which deals with the characteristic 0 case) to the case where the characteristic of the base field is not 2 or 3. This is a joint work with Kohei Fukuda.

Speaker: Yasuhiro Terakado (University of Tokyo)
Title: Determinant of l-adic cohomology of a double covering of a projective space and the discriminant of the ramification divisor
Abstract: The determinant of the Galois action on the ℓ-adic cohomology of the middle degree of a double covering of a projective space of even dimension defines a quadratic character of the absolute Galois group of the base field. In this talk, we show that the discriminant of the defining polynomial of the ramification divisor of the double covering allows us to compute the character.

Speaker: Kotaro Sugahara (Kyushu University)
Title: Arithmetic Cohomology Groups
Abstract: Parshin, Beilinson and Huber construct adelic cohomology groups for quasi-coherent sheaves

on Noetherian shchemes (especially, on algebraic varieties), and show that these correspond to cohomology groups for quasi-coherent sheaves. In this talk, we introduce adelic cohomology groups for quasi-coherent sheaves on arithmetic varieties and give some fundamental properties of those for arithmetic surfaces. This is a joint work with Prof. Lin Weng.

Speaker: Takao Yamazaki (Tohoku University)
Title: p-adic soliton theory and torsion points on Jacobian varieties
Abstract: Anderson introduced a p-adic version of soliton theory and applied it to an arithmetic problem related to Manin-Mumford conjecture. He estimated the number of p-torsion points on the theta divisor of a certain curve. We evolve his theory further and estimate the number of p^n-torsion points on the theta divisor for more general curves. (Joint work with S. Kobayashi.)

December 12th (Thu)

Speaker: Naoki Imai (University of Tokyo)
Title: The p-adic Langlands correspondence II (Survey Talk)
Abstract: In this survey talk, I talk about a compatibility of the global Langlands correspondence and the p-adic Langlands correspondence for GL_2 after Emerton. I'm planning to talk also about its application to the Fontaine-Mazur conjecture.

Speaker: Wataru Kai (University of Tokyo)
Title: A p-adic exponential map for the Picard group and its application to the Albanese map
Abstract: We define an exponential map from the first cohomology group of the structure sheaf to the Picard group of a proper flat scheme over a complete DVR of characteristic $(0, p)$. To be precise, it is an isomorphism between subgroups of each member. It is an analogue of the classical one defined in complex geometry. This exponential map is then applied to prove a surjectivity property concerning the Albanese map of a smooth projective variety over a complete DVF.

Speaker: Hiroyasu Miyazaki (University of Tokyo)
Title: Special values of zeta functions of varieties over finite fields via higher Chow groups
Abstract: We give a formula which calculates special values at non-positive integers of zeta functions of varieties over a finite field by using higher Chow groups. To be precise, we define functorial maps from higher Chow groups to weight homology groups, and prove that special values can be described up to sign as alternating products of ratios of orders of kernels and cokernels of the maps. Here, weight homology group is an invariant which measures singularity of varieties. Work in this direction was started by K. Kato. He proved the formula for special values at 0 in two dimensional cases. Moreover, M. Kerz and S. Saito showed the formula for special values at 0 when varieties are proper and smooth. In the construction of the maps and in the proof of the formula, the notion of weight complex, which was firstly introduced by H. Gillet and C. Soule, plays an important role.

Speaker: Masatoshi Suzuki (Tokyo Institute of Technology)
Title: An inverse problem for canonical systems and its applications
Abstract: A canonical system is a first-order system of ordinary differential equations parameterized by all complex numbers. A number of pairs of the second-order differential equations and systems of first-order differential equations are reduced to canonical systems. It is known that the solution of a canonical system generates an entire function of the Pólya class. Therefore, one conceivable inverse problem is to

recover a canonical system from a given entire function of the Pólya class. This type of inverse problem was solved in general by de Branges in 1960s. However his results are often not enough to investigate the Hamiltonian of a recovered canonical system. In this talk, we present an explicit way to recover the Hamiltonian of a canonical system from a given exponential polynomial belonging to the Hermite-Biehler class. After that, we apply it to study distributions of roots of self-reciprocal polynomials and zeros of zeta functions.

Speaker: Yu Yasufuku (Nihon University)
Title: Arithmetic Properties of Coordinates of Orbit Points
Abstract: Given an algebraic variety X and a self-map $f : X \to X$ defined over a number field and a rational point P on X, we define the orbit of P under f to be $\{P, f(P), f(f(P)), \ldots\}$. Silverman proved that orbits under a morphism f on $\mathbb{P}^1$ only contain finitely many S-integers unless f satisfies a special ramification property, and this can be generalized to maps on $\mathbb{P}^n$, albeit assuming a very deep diophantine conjecture of Vojta. In this talk, we will briefly summarize these results, and then discuss several explicit examples of morphisms and rational maps on $\mathbb{P}^n$ for which S-integral points in orbits can be analyzed unconditionally. A connection with a dynamical Mordell–Lang problem will also be mentioned.

Speaker: Noriyuki Abe (Hokkaido University)
Title: A classification of irreducible admissible modulo p representations of reductive p-adic groups
Abstract: We describe a classification of irreducible admissible modulo p representations (representations over a field of characteristic p) of reductive p-adic group in terms of supercuspidal representations. This generalizes the result of Barthel-Livne for GL(2), Herzig for GL(n) and my previous work for split groups. Our motivation is the hypothetical modulo p Langlands correspondence. This is a joint work with G. Henniart, F. Herzig and M.-F. Vigneras.

December 13th (Fri)

Speaker: Masato Kurihara (Keio University)
Title: Rubin-Stark conjecture and Rubin-Stark elements (Survey Talk)
Abstract: The Stark conjecture and the Rubin-Stark conjecture which generalizes a refined version of the Stark conjecture in the abelian case predict the existence of some algebraic elements related to the zeta values (when the base field is the rational number field, they are related to cyclotomic units and Gauss sums). I will survey the Rubin-Stark conjecture. I will also talk on the relation with Euler systems, and on a noncommutative version of the conjecture if I have time.

Speaker: Takamichi Sano (Keio University)
Title: A new conjecture for Rubin-Stark elements and its applications
Abstract: In this talk, we present a new conjecture concerning Rubin-Stark elements. We prove that the equivariant Tamagawa number conjecture (for Tate motives) implies our conjecture. As applications, we give another proof of Darmon's conjecture on cyclotomic units, which was recently solved by Mazur and Rubin via a method of Kolyvagin systems, and proofs of several conjectures of Gross for abelian fields.

Speaker: Sachio Ohkawa (University of Tokyo)
Title: On logarithmic nonabelian Hodge theory of higher level in characteristic p
Abstract: Ogus and Vologodsky constructed a positive characteristic analogue of Simpson's correspondence over the complex number field between local systems and Higgs bundles. One of the key technical

results for their construction is the fact that the sheaf of differential operators of level 0 forms an Azumaya algebra over its center. In this talk, we generalize the Azumaya nature of the sheaf of differential operators to the case of log differential operators of higher level. We also give some splitting modules of this Azumaya algebra to get the equivalence between certain $\mathcal{D}$-modules and certain Higgs modules.

Speaker: Yuya Matsumoto (University of Tokyo)
Title: Good reduction criterion for K3 surfaces
Abstract: The Néron–Ogg–Shafarevich criterion states that an abelian variety (over a complete discrete valuation field) has good reduction if and only if the Galois action on the (l-adic) Tate module satisfies a certain property. In the complex case, there is a similar criterion for K3 surfaces, which can be deduced from a result of Kulikov on semistable reduction of complex K3 surfaces. In this talk we show a similar criterion for K3 surfaces (using the l-adic cohomology in place of the l-adic Tate module) in the mixed characteristic case.

Speaker: Keisuke Arai (Tokyo Denki University)
Title: Non-existence of points rational over number fields on Shimura curves
Abstract: I will talk about the non-existence of points rational over number fields on Shimura curves with/without level structure. The result for Shimura curves with level structure is an analogue of that for rational points on the modular curve $X_0(p)$. From the result for Shimura curves without level structure, we obtain a counterexample to the Hasse principle. Both of these new results can be applied to number fields of higher degree, though the known results were restricted to number fields of degree ≤ 2.

Speaker: Ahmed Abbes（IHES/CNRS）
Title: The p-adic Simpson correspondence
Abstract: In 2005, Gerd Faltings laid the foundations of a correspondence aimed at describing all p-adic representations of the geometric fundamental group of a smooth algebraic variety over a p-adic field in terms of linear algebra, specifically of Higgs bundles. His construction is for p-adic Hodge theory what the complex Simpson correspondence is for classical Hodge theory. I will present a new approach for the p-adic Simpson correspondence, closely related to Faltings's original approach, and inspired by the work of Ogus and Vologodsky on an analogue in characteristic $p > 0$. This is a joint work with Michel Gros.

Program Committee: Tadashi Ochiai (Osaka University), Takeshi Tsuji (University of Tokyo), Iwao Kimura (University of Toyama)