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楕円モジュラーの場合のlocal newform

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§1. Intro

Ref (GL(2))

[1]. W. Casselman, Math. Ann. 201 1973

[2]. H. Shimizu, 保型関数 III §11.5

[3]. R. Schmidt, J. Ramanujan Math. Soc. 17, 2002

Application

Auto. rep $\longleftrightarrow$ Auto. forms

GL(2)

Jacquet-Langlands corr. (auto. rep)

$\longrightarrow$ Eichler, Shimizu corr. (auto forms)

$GS_p(4)$

- Schmidt (Saito-Kurokawa lifting)
- Sorensen
- Roberts, Weissauer, Yoshida type.

$\longrightarrow$ hol paramodular forms$/\mathbb{Q}$ are not Yoshida type (trivial central char).

§2. Casselman [1]

F is a non-arch. local field, char $F=0$

$|\ |_F$: valuation

$\mathcal{O} = \{x \in F \mid |x|_F \leq 1\}$

$\mathcal{O}^\times = \{\ \ \ \ : \ \ |x|_F = 1\}$ $\quad$ $\mathfrak{p}$ is the max. ideal of $\mathcal{O}$

ϖ_F is a prime element

$\mathfrak{p} = \varpi\mathcal{O}$ $\quad q = \#(\mathcal{O}/\mathfrak{p}) = |\varpi_F|^{-1}$

$G_F = GL_2(F)$, V is a rep space$/\mathbb{C}$

$\pi : G_F \longrightarrow GL(V)$ rep

(π, V) is an infinite dim'l irred adm. rep of G_F

$\eta : F^\times \longrightarrow \mathbb{C}^\times$ s.t. $\pi\left(\begin{pmatrix} \tau & 0 \\ 0 & \tau \end{pmatrix}\right) = \eta(\tau)\, id_V$

$$\Gamma_0(\mathfrak{p}^n) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G_{\mathcal{O}} \;\middle|\; c \in \mathfrak{p}^n \right\}$$

$n > 0$, $V_n = \left\{ x \in V \;\middle|\; \pi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) x = \eta(d) x \quad \forall \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(\mathfrak{p}^n) \right\}$

$n = 0$ $\quad V_0 = \{ x \in V \mid \pi(g)\cdot x = x,\ \forall g \in G_{\mathcal{O}} \}$

<u>Thm (Casselman)</u>

There exists an integer $n \in \mathbb{Z}_{\geq 0}$ s.t $V_n \neq \{0\}$

$C(\pi)$ is the smallest integer s.t. $V_{C(\pi)} \neq \{0\}$

Then $\dim V_{C(\pi)} = 1$

If $n \geq C(\pi)$, $V_n = \bigoplus_{i=0}^{n-C(\pi)} \pi\left(\begin{pmatrix} 1 & 0 \\ 0 & \varpi^i \end{pmatrix}\right) V_{C(\pi)}$

$\dim V_n = n - C(\pi) + 1$

the conductor of π

Proof

principal series, special $\longrightarrow$ [1] Induced rep

[2] Kirillov model

supercuspidal $\longrightarrow$ [1], [2] Kirillov model

($\eta|_{1+\mathfrak{p}^{c(\pi)}} = 1$ に注意)

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$\mu_1, \mu_2 : F^\times \longrightarrow \mathbb{C}^\times$ quasi-char.

$$B(\mu_1, \mu_2) := \left\{ \varphi : G_F \longrightarrow \mathbb{C} \text{ loc. constant. } \begin{array}{l} \varphi\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} g\right) = \mu_1(a)\mu_2(b)\left|\frac{a}{d}\right|_F^{\frac{1}{2}} \varphi(g) \\ \forall \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G_F, \ \forall g \in G_F \end{array} \right\}$$

$$(\rho(\mu_1, \mu_2)(g)\varphi)(h) := \varphi(hg) \quad \varphi \in B(\mu_1, \mu_2)$$

$$\forall g, h \in G_F$$

$(\rho(\mu_1, \mu_2), B(\mu_1, \mu_2))$ induced rep.

① $\frac{\mu_1}{\mu_2} \neq |\ |_F$ or $|\ |_F^{-1}$ $\rho(\mu_1, \mu_2)$ irred.

$\longrightarrow \pi(\mu_1, \mu_2)$ principal series

② $\frac{\mu_1}{\mu_2} = |\ |_F$ · $B(\mu_1, \mu_2)/\mathbb{C}u$ irred. $\longrightarrow \sigma(\mu_1, \mu_2)$

③ $\frac{\mu_1}{\mu_2} = |\ |_F$ $\exists$ closed subsp. irred. $\longrightarrow$: special

$$\pi(\mu_1, \mu_2) \simeq \pi(\mu_2, \mu_1)$$

$$\sigma(\mu_1, \mu_2) \simeq \sigma(\mu_2, \mu_1)$$

Cor. of Proof

(i) If π is supercuspidal, then $c(\pi) \geq 2$

(ii) $c(\pi) = 0 \iff \pi \simeq \pi(\mu_1, \mu_2)$, $\mu_i|_{\mathcal{O}^\times} = 1$

(iii') η is trivial

$c(\pi) = 1 \iff \pi \simeq St$ or ξSt

$St = \sigma(|\ |_F^{\frac{1}{2}}, |\ |_F^{-\frac{1}{2}})$

$\xi^2 = 1$, $\xi|_{\mathcal{O}^\times} = 1$, ξ is a char. of $F^\times$

<u>注意</u>

$f \in S_k^{new}(\Gamma_0(N))$ N is square-free

$\rightsquigarrow \pi_f = \bigotimes_v \pi_{f,v}$ auto rep of $PGL(2)/_{\mathbb{Q}}$

$v | N$ $\pi_{f,v} \simeq St$ or ξSt (ξ unr)

$L(s, \pi_{f,v}) = (1 - q^{-\frac{1}{2}})^{-1}$ or $(1 + q^{-\frac{1}{2}-s})^{-1}$

<u>注</u> ε-factor

$\varepsilon(s, \pi, \psi) = \underset{\text{const indep. of } s}{c} \cdot q^{(2c(\psi) - c(\pi))s}$

where $\psi : F \to \mathbb{C}^\times$ additive char.

$c(\psi)$ is the smallest integer s.t. $\psi|_{c(\psi)} = 1$

§3. Scmidt [3]

$\mu : F^\times \longrightarrow \mathbb{C}^\times$ quasi-char.

$C(\mu)$ is the smallest integer s.t. $\mu|_{1+\mathfrak{p}^{c(\mu)}} = 1$.

Thm (1) If $\pi \simeq \pi(\mu_1, \mu_2)$ then $C(\pi) = C(\mu_1) + C(\mu_2)$

(2) If $\pi \simeq \chi\, St$ (χ : unr), then $C(\pi) = 1$

(3) If $\pi \simeq \chi\, St$ and χ is ramified,

then $C(\pi) = 2\, C(\chi)$

E/F quad. ext.

ξ is a quasi-char. of $E^\times$

$\xi|_{\ker N_{E/F}} \neq$ trivial

A dihedral supercuspidal rep w_ξ is determined by E and ξ

$f(E/F)$ is the degree of the res. class field ext. of E/F ($= 1$ or 2)

$d(E/F)$ is the valuation of the disc. of E/F

$$\left(= \begin{cases} 0 & E/F \text{ unramified} \\ 1 & E/F \text{ ramified, } q \text{ is odd} \end{cases} \right.$$

Thm $C(w_\xi) = f(E/F)\, C(\xi) + d(E/F)$

i) $C(w_\xi) = 2\, C(\xi)$ if E/F is unr.

ii) $C(w_\xi) = C(\xi) + 1$ if E/F is ram. and q : odd

$GL(n)$

Jacquet, Piatetski-Shapiro, Shalika
Math. Ann. 256 (1981)